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Do ultraviolet renormalons contribute to the QCD static potential?

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We find that perturbation theory fails to describe the short range static potential as obtained from quenched lattice simulations, at least for source separations $r > (6\ \mathrm{GeV})^{-1}$. The difference between the non-perturbatively determined potential and perturbation theory at short distance is well parameterised by a linear term with a slope of approximately 1.2 GeV^2, that is significantly bigger than the string tension, $\sigma \approx 0.21$ GeV^2.

1 Introduction

The operator product expansion (OPE) constitutes the standard framework of including non-perturbative contributions to a QCD observable. Within the OPE, non-perturbative power corrections are included in the form of infrared sensitive operators of a given dimension that are accompanied by ultraviolet coefficient functions. These Wilson coefficients are in general themselves only fixed up to power corrections that can be reshuffled into higher dimensional operators without affecting physical results. This ambiguity is related to the presence of infrared renormalons [1] and also reflects the asymptotic character of the QCD perturbative series.

The powers of the correction terms within the OPE are completely determined by the symmetries of the operator that is expanded. In a standard infrared

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renormalon analysis the dimension of the lowest order renormalon contribution coincides with that of the lowest non-perturbative condensate that contributes. In the case of the running coupling this would for instance be the gluon condensate [2], such that the leading order power correction to $\alpha_s(q)$ is expected to be proportional to Λ^4/q^4. This picture has recently been challenged by the ultraviolet renormalon technique [3] and dispersion approaches [4–6]. It has been argued that in addition to the terms expected from the OPE, a further ultraviolet renormalon contribution, proportional to Λ^2/q^2 might be present. Insofar, renormalons seem to allow for a more universal classification of power-like corrections than the OPE [7]. Power corrections to α_s have been known to arise naturally in many physical schemes for quite a while [8].

Evidence of an unexpected Λ^2/q^2 power correction to the gluon condensate has been obtained in the lattice study of Ref. [9]. Recently, the running coupling from the three-gluon vertex determined in Landau gauge on the lattice has been investigated and also here evidence for a contribution proportional to Λ^2/q^2 has been reported [10].

In case of the static potential in position space, such a Λ^2/q^2 contribution to the running coupling results in a term proportional to the quark separation, r, while the leading order correction from the standard renormalon analysis depends only quadratically on r [11]. Indeed, based on various reasonable model assumptions such linear short-range contributions have been obtained [11–13].

The present letter is motivated by these latest developments and the phenomenological relevance of the short-range static QCD potential in view of calculations of the $t\bar{t}$ threshold cross-section. We summarise recent perturbative results, describe the method that we use to subtract the QCD perturbative series and, indeed, find the non-perturbative contribution to be dominated by a linear term.

2 The potential in perturbation theory

Recently, the static potential,

$$V(q) = -C_F \frac{4\pi\alpha_V(q)}{q^2}, \tag{1}$$

has been calculated to two loops by Peter [14,15]. This calculation has been independently confirmed by Schröder [16] who corrected a normalisation error of the original reference. The result reads,

$$\alpha_V(q) = \alpha_{\overline{MS}}(q)\left(1 + a_1\alpha_{\overline{MS}}(q) + a_2\alpha^2_{\overline{MS}}\right). \tag{2}$$

For the quenched approximation ($n_f = 0$) they obtain,

$$a_1 = \frac{31}{12\pi}, \quad a_2 = \left(\frac{4343}{18} + 36\pi^2 - \frac{9}{4}\pi^4 + 66\zeta(3)\right)\frac{1}{16\pi^2}. \tag{3}$$

$\zeta(3) \approx 1.202056903$ denotes the Riemann ζ-function. a_1 has already been calculated some time ago [17,18].

A Fourier transformation yields

$$V(r) = -C_F \frac{\alpha_R(1/r)}{r} \tag{4}$$

for the potential in position space with [18,15,19],

$$\alpha_R(1/r) = \alpha_V(\mu)\left(1 + \frac{\pi^2\beta_0^2}{3}\alpha_V^2\right), \quad \mu = \exp(-\gamma_E)/r. \tag{5}$$

γ_E denotes the Euler constant. The coefficients of the QCD β-function for $n_f = 0$ are given by [20],

$$\beta_0 = \frac{11}{4\pi}, \quad \beta_1 = \frac{102}{16\pi^2}, \quad \beta_2^{\overline{MS}} = \frac{2857}{128\pi^3}. \tag{6}$$

By employing a recursive lattice finite size technique [21], the ALPHA collaboration has recently obtained a value for the running coupling in quenched QCD [22],

$$\Lambda_{\overline{MS}}^{(0)} = 0.602(48)\, r_0^{-1}. \tag{7}$$

$r_0 \approx 0.5$ fm denotes the Sommer [23] scale of eq. (10) below.

We choose to parametrise the solution of the renormalisation group equation by,

$$\alpha_{\overline{MS}}(\mu) = \frac{1}{\beta_0}\left[t + d_1 \ln t + d_1^2 \frac{\ln t}{t} + \frac{d_2}{t} - \frac{d_1^3}{2}\frac{\ln^2 t}{t^2} + \frac{\beta_1\beta_2}{\beta_0^5}\frac{\ln t}{t^2}\right]^{-1} \tag{8}$$

with $t = 2\ln(\mu/\Lambda_{\overline{MS}})$, $d_1 = \frac{\beta_1}{\beta_0^2}$ and $d_2 = d_1^2 - \frac{\beta_2}{\beta_0^3}$, which is consistent to order α_s^3. The integration constant corresponds to the standard normalisation convention of the QCD Λ-parameter.

3 The lattice potential

We have collected lattice data on the static potentials from Refs. [24–27]. All these results have been obtained in the quenched approximation with Wilson action at values of the bare lattice coupling $6.0 \leq \beta = 6/g_0^2 \leq 6.8$ that correspond to lattice spacings $0.19\,r_0 > a > 0.069\,r_0 \approx 0.035$ fm. The linear lattice extents are kept at a similar size in physical units and range from $L = 16$ at $\beta = 6.0$ to $L = 48$ at $\beta = 6.8$. In all cases, we find $La > 3\,r_0 \approx 1.5$ fm, such that finite size effects are negligible [24,28].

All lattice data within this β-range can be well fitted to the Cornell ansatz,

$$aV(r) = aV_0(a) - \frac{e}{ra^{-1}} + \sigma a^2 ra^{-1} \tag{9}$$

for $r > 0.4\,r_0$, where rotational invariance is found to be restored within statistical errors. The parameter $V_0(a)$ that contains the self energy of the two static sources diverges like $-1/[a\ln(\Lambda a)]$ as the continuum limit is taken. The scale r_0 is defined by [23],

$$\left.\frac{dV(r)}{dr}r^2\right|_{r=r_0} = 1.65. \tag{10}$$

From quarkonium phenomenology one finds [23,26] $r_0 \approx 0.5$ fm. We obtain this scale from our fits to eq. (9): $r_0a^{-1} = \sqrt{(1.65-e)/(\sigma a^2)}$.

In Fig. 1, we plot the short range part of the lattice potentials obtained at different lattice spacings a_β in units of r_0. The statistical errors are all smaller than the corresponding data symbols. The normalisation convention adopted to cancel the self-energy is $V(r_0) = 0$. The first four data points measured at each β-value correspond to $r/a_\beta = 1, \sqrt{2}, \sqrt{3}, 2$, respectively. The data for $r > 2a_\beta$ all follow a universal continuous curve and rotational invariance is restored qualitatively. We take this as indication that for these distances, the continuum limit is effectively realised. For comparison, the result from the Cornell fit to $r > 0.4\,r_0$, used to determine the scale r_0, is plotted (dashed curve). Towards small r the data points lie above this curve, indicating asymptotic freedom, i.e. a weakening of the effective Coulomb coupling with the distance.

In addition, we have included the one-loop (dashed dotted curve) and two-loop (solid curve with error band) continuum expectation of eqs. (2)–(8). Note that, up to the additive self-energy constant, no free parameter can be adjusted. For better comparison with the lattice data, normalised to $V(r_0) = 0$, we have subtracted the values $0.94\,r_0^{-1}$ and $0.77\,r_0^{-1}$ from the one- and two-loop expectations, respectively. The perturbative expression has a pole at $\Lambda_{\overline{MS}}^{-1} \approx$

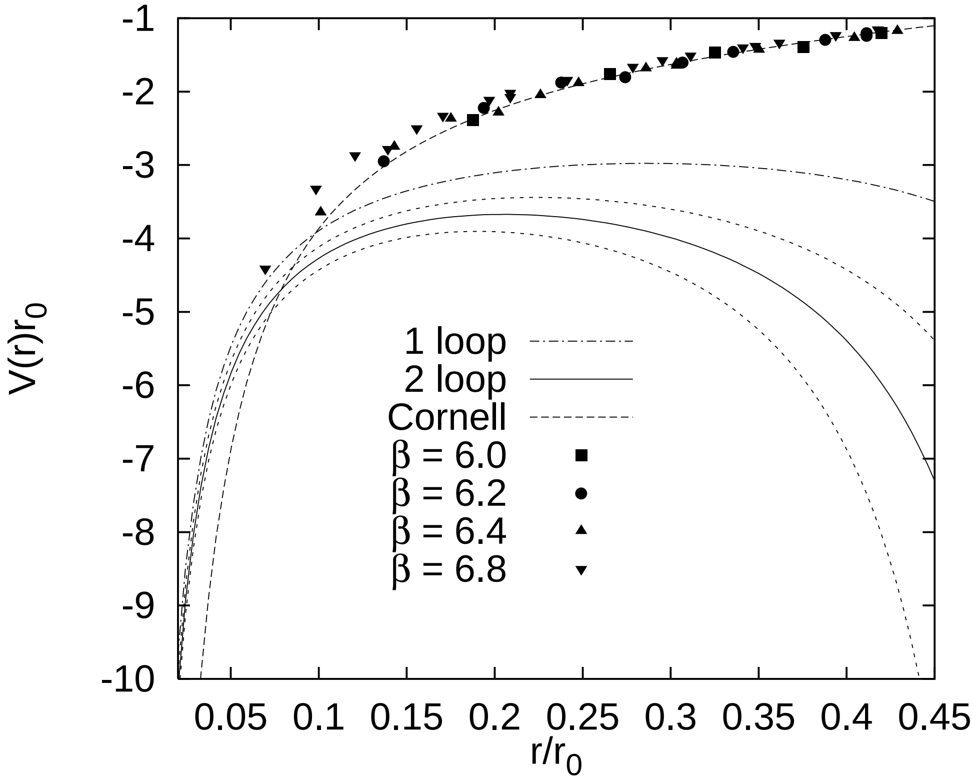

Fig. 1. The quenched lattice potential versus perturbation theory in units of $r_0 \approx 0.5$ fm. The only free parameter is the self-energy of the static sources. The lattice data has been normalised such that $V(r_0) = 0$. The values $0.94\,r_0^{-1}$ and $0.77\,r_0^{-1}$ have been subtraced from the perturbative one- and two-loop formulae, respectively, to allow for better comparison.

$1.66\,r_0 \approx (240 \text{ MeV})^{-1}$, such that we would not expect perturbation theory to reliably describe the lattice data for $r > 0.5\,r_0 \approx 0.25$ fm or so. As can be seen from the plot, even at the smallest distance at our disposal, $r \approx 0.07\,r_0 \approx (24\,\Lambda_{\overline{MS}})^{-1}$, the lattice data substantially deviates from perturbation theory; at any given distance the perturbative formula considerably under-estimates the slope dV/dr.

In Ref. [29] it has been pointed out that due to the different form of higher order corrections, depending on whether one calculates the potential in position or momentum space, numerically significant differences arise. We have convinced ourselves that in the quenched theory for $0.05\,r_0 < r < 0.4\,r_0$ this systematic uncertainty can indeed be numerically up to two times as big as the error due to the uncertainty in $\Lambda_{\overline{MS}}$ displayed in the plot. However, we found that the observed disagreement with perturbation theory cannot be repaired by fiddling around with the parametrisation. We remark that the difference between one- and two-loop perturbation theory is quite significant as indicated by the value, $\beta_2^V = \beta_2^{\overline{MS}} + a_1\beta_1 + (a_2 - a_1^2)\beta_0 \approx 3.19 \gg \beta_2^{\overline{MS}} \approx 0.72$.

In position space, the correction is even more significant: starting from eqs. (2) and (5) and the definition,

$$\alpha_R(1/r) = \alpha_{\overline{MS}}(1/r)\left[1 + c_1\alpha_{\overline{MS}}(1/r) + c_2\alpha_{\overline{MS}}^2\right], \tag{11}$$

we obtain,

$$c_1 = a_1 + 2\gamma_E\beta_0, \quad c_2 = a_2 + 4\gamma_E\beta_0 a_1 + 2\gamma_E\beta_1 + \left(4\gamma_E^2 + \frac{\pi^2}{3}\right)\beta_0^2. \tag{12}$$

This results in the numerical value, $\beta_2^R = \beta_2^{\overline{MS}} + c_1\beta_1 + (c_2 - c_1^2)\beta_0 \approx 6.70$.

4 Non-perturbative ultraviolet contributions

Given the asymptotic character of QCD perturbative series, it is always ambiguous how to subtract perturbative contributions to a given physical process in order to isolate non-perturbative terms. Bearing this in mind, we employ the procedure of subtracting the one- and two-loop result, given by eqs. (2)–(8). Needless to say, that using a different scheme or cutting off the series at a different order in α_s will in general yield somewhat different results. We would like to remark that beyond the two-loop level there appear to be difficulties in a consistent perturbative definition of the static potential [30,31].

In Figs. 2 and 3, we have subtracted the one- and two-loop continuum perturbation theory result that correspond to the central value $\Lambda_{\overline{MS}} = 0.602\, r_0^{-1}$ from our lattice data. The resulting points are compared to linear curves (solid lines) with slopes $4.6\, r_0^{-2} \approx 0.72$ GeV^2 and $7.7\, r_0^{-2} \approx 1.20$ GeV^2, respectively. We have added the error band due to the uncertainty in $\Lambda_{\overline{MS}}$ to these curves. For $r > 0.4\, r_0$ and $r > 0.3\, r_0$ for the one- and two-loop results, respectively, deviations from the linear behaviour become visible while for r values that are small in units of the lattice spacings a_β the points scatter around the interpolating curve due to lattice artefacts. However, it is evident that the dominant short range correction to perturbation theory is proportional to r.

Given the visible lattice structure at small r, we do not attempt a fit to the data but we conservatively estimate systematic uncertainties of up to 30 % on the observed slopes by allowing the interpolating curves to touch the most extreme combinations of data points that are possible. The results are 3.4 ± 1.0 and 5.7 ± 1.7 times bigger than the string tension $\sigma \approx 1.35\, r_0^{-2}$, respectively. Neither a term of the size of the string tension as suggested in Ref. [32] nor a linear short-range term $\Delta V_2(r) = \frac{18}{2\pi}\alpha_s\sigma r$ predicted by Simonov [12] can alone

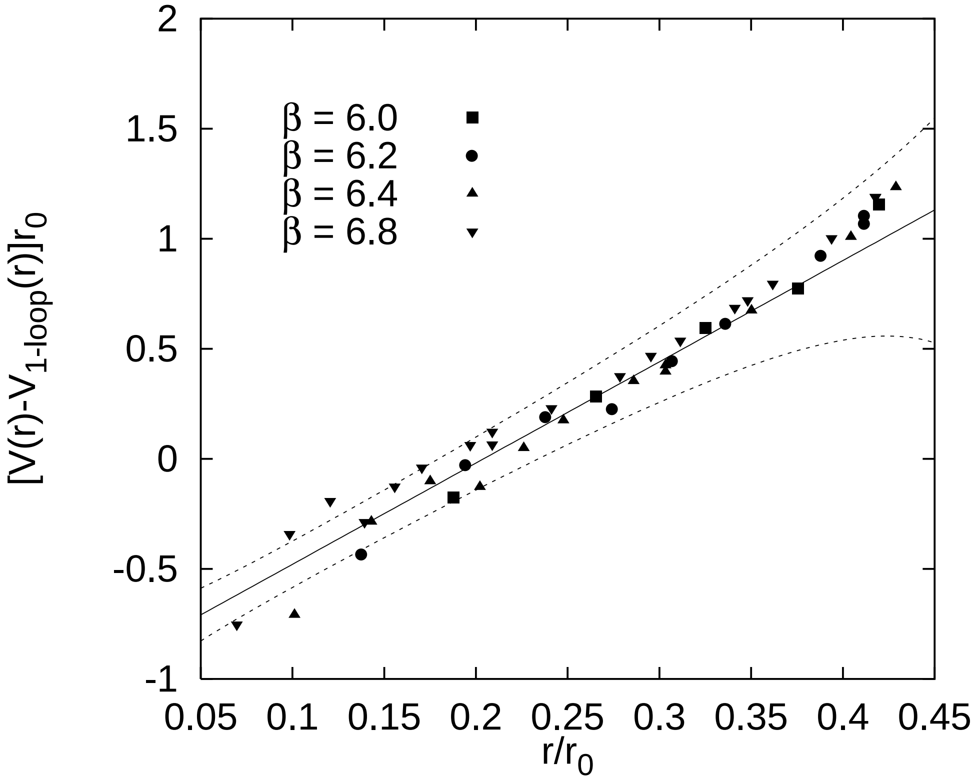

Fig. 2. The quenched lattice potential with one-loop perturbation theory subtracted, in comparison with a linear curve with slope $4.6\, r_0^{-2}$.

account for this slope. Thus, additional non-perturbative contributions must be present.

We would like to emphasize that the independent determination of the QCD Λ-parameter by the ALPHA collaboration [22] was essential for our study: it is always possible to effectively compensate part of the slope by increasing the value of the Λ-parameter that enters the perturbative formula.

We choose to parameterise the running coupling into,

$$\alpha_R(1/r) = \alpha_{R,2-loop}(1/r) + c_R r^2 \tag{13}$$

and find $c_R = -(0.90 \pm 0.27)$ GeV^2. A transformation into momentum space yields, $\alpha_V(q) = \alpha_{V,2-loop}(q) + c_V/q^2$ with $c_V = -c_R/2$. It is not *a priori* clear how to convert the result into different schemes and how to apply it to different physical observables. In fact, even the conversion to momentum space is not exact since the Fourier transformation affects higher order contributions and thus the difference between our result and perturbation theory. This can then

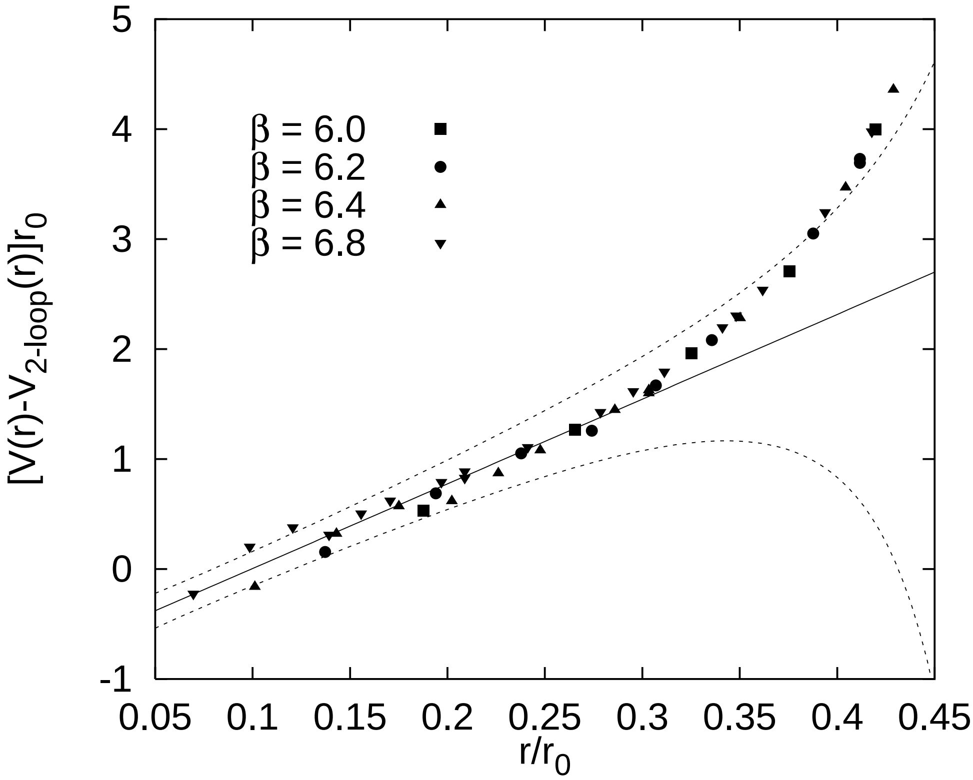

Fig. 3. The quenched lattice potential with two-loop perturbation theory subtracted, in comparison with a linear curve with slope $7.7\, r_0^{-2}$.

result in a somewhat different slope of the linear term, due to the asymptotic character of the perturbative series.

5 Conclusion

We find that perturbation theory fails to describe the short range static potential as obtained from quenched lattice simulations, at least for source separations bigger than $(6\ \mathrm{GeV})^{-1}$. The difference between the non-perturbatively determined potential and perturbation theory at short distance is well parameterised by a linear term. We quote the two-loop slope, $(1.20\pm0.36)\ \mathrm{GeV}^2$ as our final result, which is significantly bigger than the string tension $\sigma \approx 0.21\ \mathrm{GeV}^2$.

It is certainly worthwhile to repeat the present calculation using lattice actions different from the Wilson action to confirm universality of the result. A further significant reduction of the lattice spacing is at present ruled out by the size of available computers as is a similar study with inclusion of sea quarks. In order to keep the argumentation transparent, we did not attempt to correct the potential for perturbatively known tree-level or one-loop lattice artefacts. This can in principle be done to reduce the amount of violations of rotational invariance in the short distance lattice potential. Unfortunately, no two-loop lattice perturbation theory result on the potential exists so far, such that a direct study of renormalon contributions on the lattice rather than in the continuum is not yet consistently possible to order α_s^3.

It should be interesting to investigate the same question in $SU(N)$ gauge theory in three space-time dimensions (which is super-renormalisable) as well as to study the behaviour on small lattice volumes, where it is possible to further reduce the lattice spacing. As a consequence of an altered vacuum structure, confinement is lost on spatial volumes smaller than about 1 fm^3. However, contributions that are related to ultraviolet physics should still be present and could easily be disentangled from terms that arise due to infrared properties of the theory.

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References

[1] G. ‘t Hooft, in: The Whys of Subnuclear Physics, Proc. Int. School, Erice 1977, ed. A. Zichichi, (Plenum, New York, 1977); B. Lautrup, Phys. Lett. 69B, 109 (1977); A.H. Mueller, Nucl. Phys. B 250, 327 (1985); A.H. Mueller, in: QCD: 20 years later, ed. P.M. Zerwas and H.M. Kastrup, (World Scientific, Singapore, 1993); M. Beneke and V.I. Zakharov, Phys. Lett. B 312, 340 (1993).

[2] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147, 385 (1978).

[3] V.I. Zakharov, Nucl. Phys. B 385, 385 (1992); A.I. Vainshtein and V.I. Zakharov, Phys. Rev. Lett. 73, 1207 (1994).

[4] R. Akhouri and V.I. Zakharov, hep-ph/9705318; Nucl. Phys. B (Proc. Suppl.) 64, 350 (1998).

[5] G. Grunberg, hep-ph/9705290; hep-ph/9705460; JHEP 9811, 6 (1998).

[6] D.V. Shirkov and I.L. Solovtsev, Phys. Rev. Lett. 79, 1209 (1997); D.V. Shirkov, Nucl. Phys. B (Proc. Suppl.) 64, 106 (1998).

[7] See e.g. V.I. Zakharov, Prog. Theor. Phys. Suppl. 131, 107 (1998).

[8] S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. D 28, 228 (1983).

[9] G. Burgio, F. Di Renzo, G. Marchesini, E. Onofri, Phys. Lett. B 422, 219 (1998).

[10] G. Burgio, F. Di Renzo, C. Parrinello, C. Pittori, hep-ph/9808258.

[11] R. Akhouri and V.I. Zakharov, Phys. Lett. B 438, 165 (1998).

[12] Yu.A. Simonov, hep-ph/9902233.

[13] K.G. Chetyrkin, S. Narrison, V.I. Zakharov, hep-ph/9811275.

[14] M. Peter, Phys. Rev. Lett. 78, 602 (1997).

[15] M. Peter, Nucl. Phys. B 501, 471 (1997).

[16] Y. Schröder, Phys. Lett. B 447, 321 (1999).

[17] W. Fischler, Nucl. Phys. B 129, 157 (1977).

[18] A. Billoire, Phys. Lett. 92B, 343 (1980).

[19] M. Ježabek, M. Peter, Y. Sumino, Phys. Lett. B 428, 352 (1998).

[20] O.V. Tarasov, A.A. Vladimirov, A.Yu. Zharkov, Phys. Lett. 93B, 429 (1980); S.A. Larin, J.A.M. Vermaseren, Phys. Lett. B 303, 334 (1993).

[21] M. Lüscher, R. Narayanan, P. Weisz, U. Wolff, Nucl. Phys. B 384, 168 (1992); M. Lüscher, R. Sommer, P. Weisz, U. Wolff, Nucl. Phys. B 491, 323 (1997).

[22] ALPHA collaboration: S. Capitani, M. Lüscher, R. Sommer, H. Wittig, Nucl. Phys. B 544, 669 (1999).

[23] R. Sommer, Nucl. Phys. B 411, 839 (1994).

[24] G.S. Bali and K. Schilling, Phys. Rev. D 47, 661 (1993).

[25] G.S. Bali and K. Schilling, Int. Journ. Mod. Phys. C 4, 1167 (1993).

[26] G.S. Bali, K. Schilling, A. Wachter, Phys. Rev. D 56, 2566 (1997)

[27] $T\chi L$ collaboration: G.S. Bali et al. Nucl. Phys. B (Proc. Suppl.) 63, 946 (1998).

[28] G.S. Bali and K. Schilling, Phys. Rev. D 46, 2636 (1992).

[29] M. Ježabek, J.H. Kühn, M. Peter, Y. Sumino, T. Teubner, Phys. Rev. D 58, 14006 (1998).

[30] T. Appelquist, M. Dine, I.J. Muzinich, Phys. Rev. D 17, 2074 (1978).

[31] N. Brambilla, A. Pineda, J. Soto, A. Vairo, hep-ph/9903355.

[32] F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, hep-th/9812030.